Performance of Trajectory Plot for Serial Concatenation of BCH and Convolutional Codes

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***Abstract*— Nowadays, the use of digital communication has been highly demanding. The advanced communication is needed by people to communicate with each other at anywhere, anytime and any place. Therefore, the better performance of communication systems is very important. Extraordinary efforts have been done by many scientists to achieve better performances of digital communication. Over than 50 years, the researchers have been working hard to develop the state of the art of digital communication. Due to this phenomenon, this paper try to acquire well performance of trajectory using serial concatenation of coding. Here, two kinds of channel coding are applied, namely: BCH and Convolutional codes. Furthermore, there are also decoding process using BCJR algorithm which is implemented in the inner and outer decoding scheme for both codes. In addition, several impacts of interleaved in the EXIT chart are going to present using three types of different length of interleaver. As a result, using quite long of interleaver length, is going to produce well-match the trajectory plot and also the EXIT chart. Lastly, outer codes is well-shown by applying a BCH code for decoding process, while Convolutional code is quite good for inner codes.**

***Keywords*— Serial concatenation, BCH, Convolutional codes, trajector, EXIT chart.**

I. INTRODUCTION

In digital communications, channel coding takes a huge part in communication systems in order to create a better life in deep. The history of channel coding was started in 1948 by Shannon [1] that produced coding scheme in order to approach of channel capacity limit. The research on channel capacity still continued until 1993, which was invented of turbo code [2]. Moreover, concatenated code has been introduced in [3]. Theoretically, it produced binary linear block code. The idea behind this, there is the inner and outer codes which are used by encoder and decoder.

There are considerable works have been investigated on the model of concatenated codes in order to acquire of coding capacity and also to decrease transmission error in a digital communication system. The investigation of the convergence behaviour of iterative decoding had been started in [4]. They had proposed that Low-Density Parity-Check (LDPC) is used to calculate the convergence threshold. Furthermore, the measurement of convergence of iterative decoders related to signal to noise ratio (SNR) also investigated in [5]. Interestingly, there is a new method to measure the convergence of behaviour of iterative decoding, known as Extrinsic Information Transfer (EXIT) chart. Then, serial concatenation of Reed Solomon (RS) codes with Kite codes was investigated in [6]. Serial concatenation of RS codes with Kite codes was studied that proposed a simulation-based optimization method employing Gaussian Approximations (GA). Then, It was followed by [7] that investigated the performance analysis for concatenated coding schemes with efficient modulation techniques.

This paper proposes the performance of trajectory plot using serial concatenation of BCH and Convolutional codes. Computer simulation is applied using Matlab programming in order to gain the trajectory in the EXIT chart. In the simulation, BCJR algorithm is also used by the decoding process. There two types of decoding process that are implemented outer and inner decoding schemes for BCH and Convolutional Codes. Based on [8], Convolutional code may be matched to be applied as inner decoding scheme because it has an open EXIT chart tunenel at quite low SNR and also it approches the channel capacity. While, outer decoding process are going to be applied in this research. Then, trajectory plot of the concatenation between the EXIT function of inner CC and the EXIT function of outer BCH is discussed. Moreover, the effect of interleaver in the EXIT chart is discussed, which are using three different length of interleavers such as: 100, 10.00 and 10.000, respectively. In addition, in the simulation, there are going to run five iterations, such 1, 2, 4, 8, and 16 iterations and also five number of frames (1 to 5).

This paper is organized as follows: To begin with, there is a short introduction to research method in Section II. Then, followed by algorithm used, system parameters, and also the model of BCH and Convolutional codes that will be used in the simulation in Section III. In Section IV, Result and analysis are going to discuss briefly, related to analysis of serial concatenation and trajectory plot of BCH and Convolutional codes, the effect of interleaver length and also the EXIT chart. Finally, some conclusions will be described briefly on the results obtained in this simulation.

II. RESEARCH METHOD

In this paper, several algorithms are considered to applied in the computer simulation. To begin with, BCH codes and Convolutional codes decoding process are done by using BCJR algorithm [9]. Here, the emerge of BCJR algorithm can be implemented in the iterative decoding. This is because the BCJR algorithm can supply an effective method of optimal decoding of linear code for minimizing symbol error rate [10]. It tries to decrease the bit error probability in the decoding process indeed. This paper discussed two-channel coding namely: BCH and Convolutional codes. The implementation of the BCJR for BCH and Convolutional Codes are quite different, although, the process of the BCJR algorithm shows quite similar way. In addition, there will be The EXIT chart, which is used to visualize the characteristic of iterative exchange of extrinsic information between serially concatenated decoders [11].

III. Simulation System

A. Decoding Process

Concatenation code is defined as the multilevel coding structure scheme which is used to reduce gradually the probability of error [12]. Serial concatenated codes are composed by a cascade of an outer, an interleaver which is used to transpose the outer code words bits, and an inner encoder [13].

Furthermore, there is the soft information, which is applied in the BCJR algorithm known as LLRs (logarithmic Likelihood Ratios). Usually, it is used in order to maximise the probability that shown as follow:

$$L(bits) = \ln\left(\frac{P(bit=0)}{P(bit=1)}\right) \tag{1}$$

where *L(bits)* represent the LLRs, *P(bit=0)* and *P(bit=1)* express the probability bits which has the value logic 1 (one) or 0 (zero).

In addition, briefly the BCJR process will be represented, which input for BCJR algorithm is a priori information, while a posteriori is known as an output. The algorithm also has two kind of basic recursions such as forward and backward recursion. The formula for forward recursion is determined as follow:

$$\alpha[S_{(i,ii)}] = \sum_{T\in to((S_{(i,ii)})} \gamma(T).\alpha[fr(T)] \tag{2}$$

where $to((S_{(i,ii)})$ is the set of all transition, which is combined to the state $\alpha(S_{(i,ii)})$, $fr(T)$ is illustrated the state, which the T transition is come from.

In the backward recursion, the process is quite the same as forward style. state $S_{(i,o)}$ is beta value $\beta(S_{(i,ii)})$ [8], which is set for particular state $(S_{(i,ii)})$. The presentation of beta's value is shown bellow:

$$\beta[S_{(i,ii)}] = \sum_{T\in fr((S_{(i,ii)})} \gamma(T).\beta[to(T)] \tag{3}$$

B. Decoding process for Convolutional Code

Generally, in the convolutional code encoder process, there is a k bit information is fed into the encoder in each clock cycle. Moreover, the convolutional code encoder contains several parameters, such as: memory element, the shift registers current states S_0 and S_1, the shift register current states s_0^+ and s_1^+, the incoming bits b, and the n output bits of V_1 and V_2.

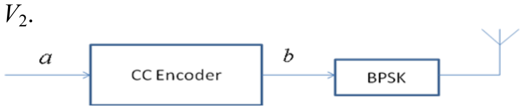

Fig. 1. Convolutional code's encoder.

Figures 2 and 3 are the result from [14], which are going to be applied in the simulation.

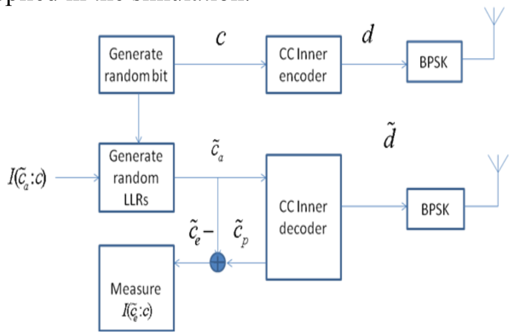

Fig. 2. Inner scheme for convolutional code.

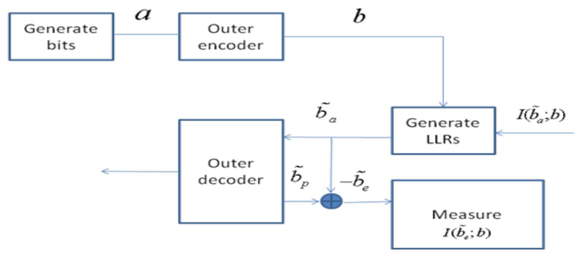

Fig. 3. Outer decoding scheme for convolutional code.

C. Decoding Process for BCH

In this paper, there are two types of BCH codes, which are shown in the Table 1. Practically, the generator polynomial for each type of BCH code can be seen in the figure below.

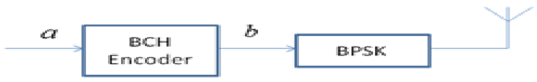

Fig. 4. Example of BCH encoder

Figure 5 describes the scheme of inner decoding of BCH (7, 4, 3) [16].

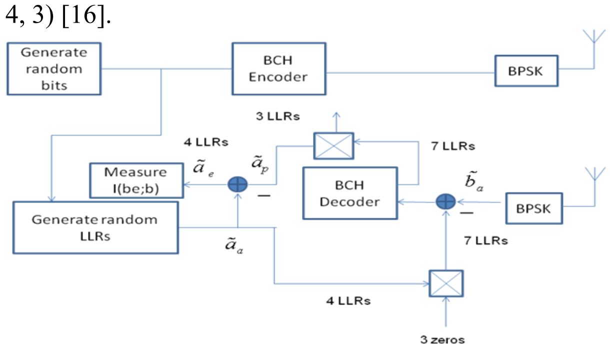

Fig. 5. Inner decoding scheme of BCH (7, 4,3).

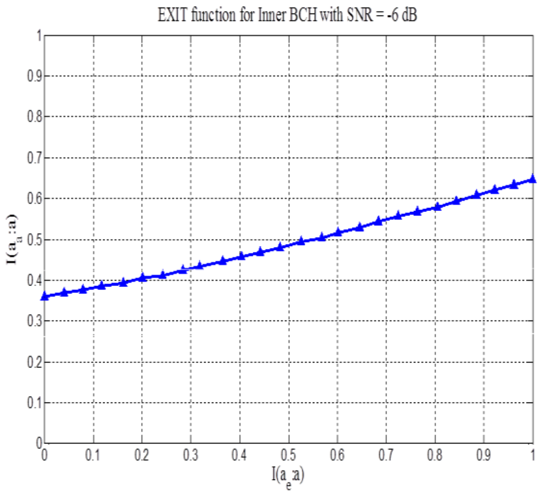

Fig. 6. The EXIT function of inner decoding of BCH (7, 4, 3) using SNR=-5 dB.

Figure 6 shows the EXIT function of iner decodding of BCH (7,4,3), which is using SNR of 5 dB. The y axis is started at 0.35 and then it is end up at 0.65 dB. It is reasonanble to say that inner decoding of BCH is not quite good implemented for the EXIT chart because it is not started at point of zero and it is not finished at point 1.

2). Outer Scheme of BCH

As highlighted, Figure 8 describes the inverted exit function of BCH (7, 4, 3). It can be seen that the y axis expresses the a priori mutual information of I (ba; b), while the x axis determines the extrinsic mutual information I(be; b). The area beneath EXIT function is 0.56862. This result is correct because the area beneath the EXIT function is quite similar to coding rate (R_{out}) which is 4 (four) divided by 7 (seven) is equal to 0.5714. The resultants of the EXIT functions are quite similar to each other as well [17].

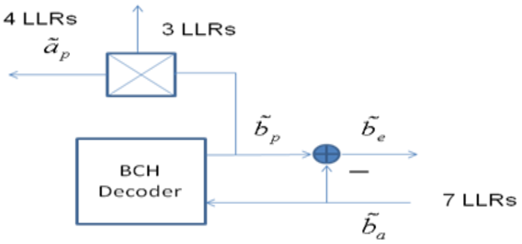

Fig. 7. Outer decoding scheme of BCH (7,4,3).

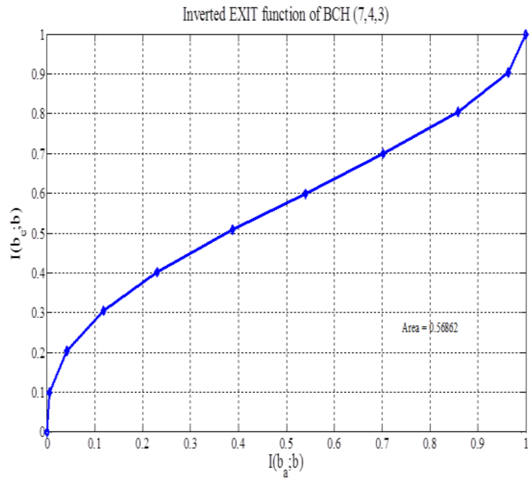

Fig. 8. The EXIT function of outer BCH (7, 4, 3) code.

D. Serial Concatenation and Trajectory plot of BCH codes and Convolutional codes

A serially concatenation codes are composed by a cascade of an outer, an interleaver which is used to transpose the outer code words bits, and an inner encoder [18]. In this paper, the serial concatenation is used BCH codes as an outer encoder and Convolutional codes as an inner encoder in the transmitter. Obviously, the process of serial concatenation will be expressed by the Figure. 9. To begin with, in the transmitter part, the length of interleaver is fed into the outer encoder BCH and then, the output of this will be created by an interleaver in order to pass it to the BPSK modulator. There are two BPSK signal will be sent to the receiver over an AWGN channel.

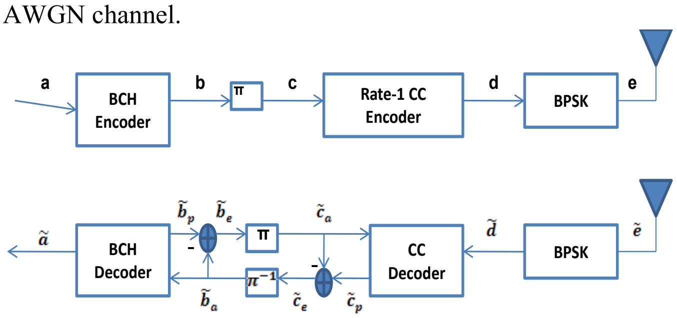

Fig. 9. Serial concatenation of BCH and CC.

On the other hand, in the receiver part, there is a priori encoded LLRs inner which is fed into a BPSK modulator. However, for the first iteration, a priori uncoded LLRs is equal to zeros. The next step is that there is a CC decoder which is consisted of a priori uncoded LLRs and a priori encoded LLRs inner decoding. In order to obtain new information, the extrinsic of uncoded LLRS inner is acquired by subtracting the a posteriori uncoded LLRs and a priori uncoded LLRs. Indeed, this result will be input for the BCH outer decoder as a priori encoded LLRs after de-interleaving them. It is followed by doing the outer BCH decoding. Lastly, extrinsic uncoded LLRs is obtained by subtracting a posteriori uncoded LLRs of outer BCH decoder with a priori encoded LLRs of the outer

BCH decoder. This process will be generated again and again to accomplish the iteration of the concatenation.

E. Simulation Parameters

The parameters for both BCH and Convolutional codes that are used in the simulation can be shown in the Table I.

TABLE I. SIMULATION PARAMETERS.

Parameter	Remarks
Coding	- BCH (7,4,3) - BCH (31,26, 3)
	CC1 (rate 1) Non-Systematic & Recursive
Interleaver Length	100, 1000 and 10000
Algorithm	BCJR and LLRs
Decoder	- Outer: BCH - Inner: Convolutional codes
Number of Frame	1 to 5
Iteration	1, 2, 4, 8 and 16

IV. RESULTS AND DISCUSSIONS

A. Trajectory plot of the concatenation between the EXIT function of inner CC and the EXIT function of outer BCH.

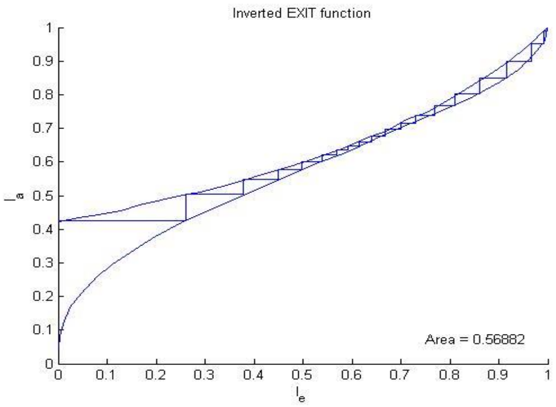

Fig. 10. Trajectory plots of BCH code and Convolutional code using BPSK modulation over AWGN channelEnergy consumption by varying packet sizes.

Figure 10 reveals the trajectory plot of the concatenation between the EXIT function of inner Convolutional Codes and the EXIT function of outer BCH. Trajectory EXIT chart is determined the combination of the extrinsic mutual information transfer with the joint of the EXIT function between BCH's outer and CC's inner. In this simulation, there are five numbers of iterations that have been implemented, such 1, 2, 4, 8 and 16 iterations, respectively. However, Figure 10 only demonstrate 1 (one) iteration as an example.

B. The effects of the Interleaver in the EXIT chart

The exchange of extrinsic information between BCH outer decoding and Convolutional inner code can be envisaged by trajectory plot [17]. In simply, the performance of the interleaver is useful in the iterative decoding. This is because of it has a massive effect on the performance of the code.

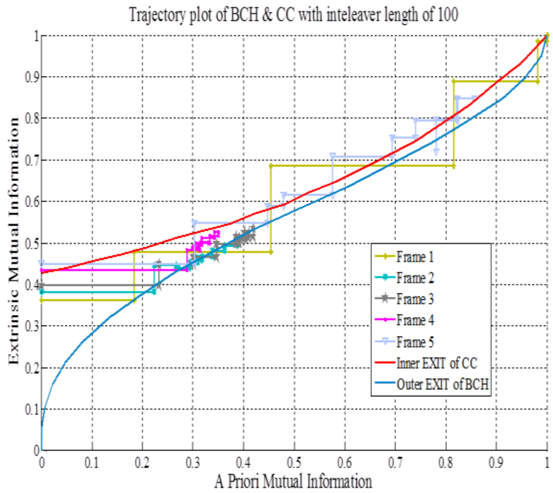

Fig. 11. Trajectory plot of BCH and Convolutional code using 100 interleaver length.

Based on the simulation result given as in Figs. 11, 12 and 13, respectively, the three of figures clearly display the trajectory plots of the outer BCH(31, 26, 3) codes and the inner Convolutional code (CC1) using three different length of interleavers, such as: 100, 1000 and 10000, respectively. The number of frames is similar to the three of figure which starts from 1 frame to 5 frames. However, the trend of the trajectories plot clearly expresses quite different performance.

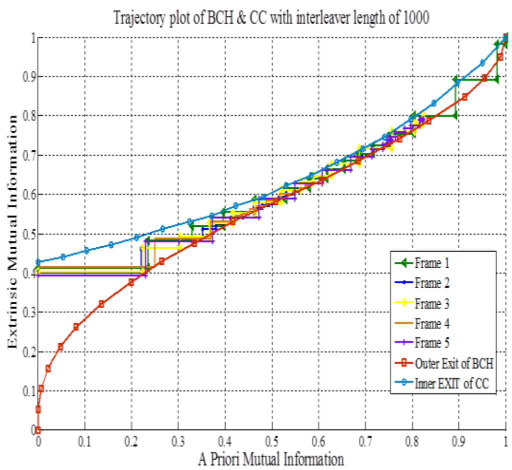

Fig. 12. Trajectory plot of BCH and Convolutional code using 1.000 interleaver length.

Figure 11 illustrates the trajectory plot of the outer BCH and the inner Convolutional codes using 100 interleaver lengths. The horizontal axis denotes as extrinsic mutual information, while a priori mutual information is determined by the vertical axis. As can be seen from the Fig. 11, each trajectory line is plotted by different color. It displays a different number of frames for plot. Actually, there is only frame 1 that can reach the (1, 1) point of the EXIT chart, while the others frames are not so lucky. They are still thriving to reach the end of the top with many iterations.

Turning to Figure 12, the trajectory plots seem to be quite better than that of Fig. 10. The number of interleaver lengths is increased 10 times, from 100 to 1.000 interleaver length. It will bring quite better trend of the trajectory plot for the outer BCH and the inner Convolutional code. After doing five (5) iterations, the trajectory lines quite fit and match the EXIT chart due to increasing the number of interleaver length to 10 times. However, seemingly, there are still number of frames showing unlucky change to reach (1, 1) point at the end of the EXIT chart.

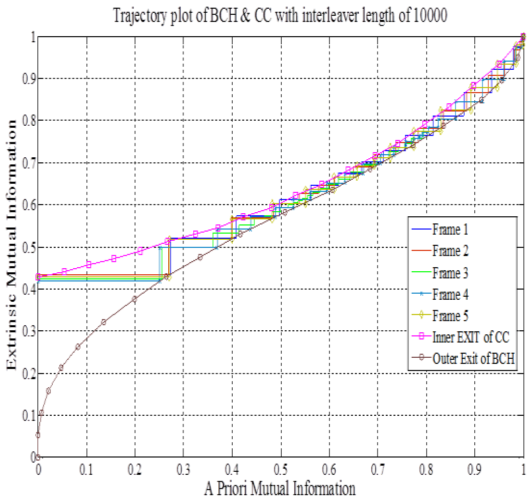

Fig. 13. Trajectory plot of BCH and Convolutional code using 10.000 interleaver length.

Interestingly, the trajectory plot using 10.000 interleaver length in the figure 13 really successfully reach the (1, 1) point of the EXIT chart. It is noticeable to say that all of the frames from 1 to 5 absolutely can achieve the (1, 1) point at the end of the EXIT chart. Furthermore, for the five frames number have the same change passing the narrow channel of the EXIT chart. It can be noticed here that a relatively short interleaver are going to produce a mismatch between the staircase trajectory and the EXIT functions, while quite long interleaver lengths are expected conversely with short interleaver length's.

V. CONCLUSIONS

This paper has been proposed the performance of the trajectory plot of serial concatenation of BCH and convolutional codes. The BCH code is well implemented as an outer code for decoding process because it has shown quite good performance of the EXIT chart. However, inner code for decoding process has been properly expressed by convolutional code, particularly CC1 (rate 1), which is kind of recursive code. This is because the EXIT function of inner decoding for Convolutional code has quite the same performance as the BPSK modulation scheme in the channel capacity measurement. Lastly, the impacts of interlever lengths really affect the trajectory plot of the EXIT chart. Based on the simulation result, if a short interleaver length ($I \approx 100$) is applied, there is a mismatch between the staircase of the iterative decoding trajectory and the EXIT chart function. While, the implementation of long interleaver length ($I \approx 10.000$) is suitable to match the trajectory plot and the EXIT chart.

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