Base Station Cooperation with Noisy Analog Channel Feedback: A Large System Analysis

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***Abstract*—Channel state information (CSI) at the base stations (BSs) can significantly increase the spectral efficiency in single and multi-cell broadcast channels. Assuming the users learn their direct and interfering channels, they can feed back this information to the BSs over the uplink. The BSs then form channel estimates which they use to design their transmission scheme. Clearly, the quality of these estimates affects system performance.**

In this paper, we study limited feedback in a two-cell MIMO broadcast channel. For a fast transfer of CSI, we consider the analog feedback scheme where the users send their unquantized and uncoded CSI over the uplink channels. In this context, given a fixed user's transmit power, we investigate how a user should optimally allocate this power to feed back the direct and interfering CSI for two types of base station cooperation schemes, namely, network MIMO and coordinated beamforming. We focus on regularized channel inversion precoding structures and perform our analysis in the large systems limit in which the number of users per cell (K) and the number of antennas per BS (N) tend to infinity with their ratio $\beta = K/N$ held fixed.

I. Introduction

The main challenge that limits the spectral efficiency in the downlink of multi-cell networks, besides intra-cell interference, is the inter-cell interference. The conventional approach to mitigate this interference is to use spatial reuse of spectral resource such as frequency and time [1]. The move towards aggressive frequency or time reuse will cause the networks to be interference limited especially for the users at the cell-edge. The current view is to mitigate the inter-cell interference through base station (BS) cooperation. Within this scheme, the BSs share the control signal, channel state information and data symbols for all users via a central processing unit or wired backhaul links [2]. MIMO cooperation schemes are also known to provide a significant increase in spectral efficiency compared to conventional cellular networks [1, and references therein]. In the *network MIMO* setup, sometimes also called multi-cell processing (MCP), the BSs fully cooperate and share both the channel state information (CSI) and transmission data.

Since in practical scenarios, the backhaul links have limited capacity, partial cooperation schemes are of interest. One example is the *interference coordination* scheme in which only CSI (including direct and interfering channels) is exchanged amongst base stations [1]. Several works have addressed coordinated power control and beamforming schemes to improve the spectral efficiency in interference-limited downlink multi-cell networks. See [1] and references therein for details.

For different levels of cooperation, the work in [3] investigated an optimization problem to minimize the total downlink transmit power while satisfying a specified SINR target. The authors derived the optimal transmit power, beamforming vectors, cell loading and achieved SINR for those different co-operation schemes in a symmetric two-cell network using large system analysis. It is also mentioned in [3] that the optimal beamforming vectors have a structure related to regularized channel inversion (RCI).

All aforementioned works assumed perfect downlink CSI available at the BSs which is hard to achieve in practice. For the interference coordination scheme, [2] studied the limited feedback (via random vector quantization (RVQ)) in an infinite Wyner cellular model using generalized eigenvector beamforming. An optimal bit partitioning strategy for direct and interfering channels was also derived. The authors in [4] took into account both CSI training and feedback in analyzing the performance of what they called inter-cell interference cancellation (ICIC) in the interference coordination setting. They also presented training optimization and feedback optimization for both analog and digital feedback (RVQ). In feedback optimization, the optimal power and bits allocations to feedback the direct and interfering channels in analog and digital feedback were investigated. However, in both [2] and [4], only one user per cell was served.

In this paper, we extend the work in [3] by investigating analog feedback optimization [4] for network MIMO and coordinated beamforming (CBf) setups. We consider a *symmetric* two-cell MISO network where the base stations have multiple antennas and the users are equipped with single antenna. We assume that the users in each cell know their own channel perfectly; they feed back this information through the uplink channel and the base stations jointly form their channel estimates. Several users are simultaneously active in each cell so that users experience both intra- and inter-cell interference. First, under the considered analog feedback model, we derive SINR expressions for both network MIMO and CBf schemes in the large systems limit where the number of antennas at BSs and the number of users in each cell go to infinity with their ratio kept fixed: *this is indicative of the average performance for even finite numbers of antennas*. We then derive an optimal transmit power allocation for the users to feed back their direct and cross channel information. Our analysis shows that

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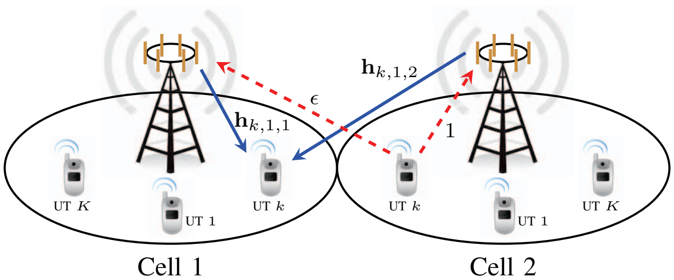


Fig. 1. System model

for cross channel gains ϵ below a certain threshold which depends on the channel estimate quality, it is better for the BSs to learn only the direct channel and perform single-cell processing (SCP). While in the perfect CSI case, network MIMO performance improves with increasing ϵ, in the limited feedback case, this only occurs for ϵ above a certain threshold.

II. System Model

We consider a symmetric two-cell broadcast channel, as shown in Figure 1, where each cell has K single antenna users and a base station equipped with N antennas. The channel between user k in cell j and the BS in cell i is denoted by row vector $\mathbf{h}_{k,j,i}$ where $\mathbf{h}_{k,j,j} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ and $\mathbf{h}_{k,j,\bar{j}} \sim \mathcal{CN}(\mathbf{0}, \epsilon\mathbf{I}_N)$, for $j = 1, 2$ and $\bar{j} = \mod(j, 2) + 1$. We refer to the $\mathbf{h}_{k,j,j}$ as direct channels and $\mathbf{h}_{k,j,\bar{j}}$ as cross or "interfering" channels. We find it useful to group these into a single channel vector $\mathbf{h}_{k,j} = [\mathbf{h}_{k,j,1}\ \mathbf{h}_{k,j,2}]$.

We consider an FDD system and assume that the users to have perfect knowledge of their downlink channels, $\mathbf{h}_{k,j,j}$ and $\mathbf{h}_{k,j,\bar{j}}$. Each user feeds back the channel information to the direct BS and neighboring BS through the corresponding uplink channels. The BSs estimate or recover these channel states and use them to construct the precoder.

The received signal of user k in cell j can be written as

$$y_{k,j} = \mathbf{h}_{k,j,1}\mathbf{x}_1 + \mathbf{h}_{k,j,2}\mathbf{x}_2 + n_{k,j}$$

where $\mathbf{x}_i \in \mathbb{C}^N$, $i = 1, 2$ is the transmitted data from BS i, and $n_{k,j} \sim \mathcal{CN}(0, \sigma_d^2)$ is the noise at the user's receiver. The transmitted data $\mathbf{x}_i$ depends on the level of cooperation assumed, and will be described in more details in Sections II-B and II-C: we restrict ourselves to linear precoding schemes, more specifically regularized zero-forcing approaches. We assume each BS's transmission is subject to a power constraint $\mathbb{E}\left[\|\mathbf{x}_i\|^2\right] = P_i$. In the network MIMO case, we relax this constraint to a sum power constraint so that $\mathbb{E}\left[\|\mathbf{x}\|^2\right] = \sum_{i=1}^2 P_i = P_t$. In the analysis, we assume $P_1 = P_2 = P$ and denote $\gamma_d = P/\sigma_d^2$.

A. CSI Feedback through AWGN Channel

As already mentioned, in practical scenarios, CSI at the BSs is normally obtained through feedback from the users.

In this paper we consider the *analog feedback* scheme, proposed in [5], where each user feeds back the CSI to the base stations using linear analog modulation. Since we skip quantizing and coding the channel information, we can convey this information very rapidly [5]. We also consider a simple uplink channel model, an AWGN channel. A more realistic multiple access (MAC) uplink channel model will be a subject for future investigation. Each user in cell j feeds back its CSI $\mathbf{h}_{k,j}$ orthogonally (in time). Since each user has to transmit $2N$ symbols (its channel coefficients), it needs $2\kappa N$ channel uses to feed back the CSI, where $\kappa \geq 1$. User k in cell j sends

$$\mathbf{h}_{k,j}\mathbf{\Lambda}_j^{\frac{1}{2}}, \tag{1}$$

where $\mathbf{\Lambda}_j$ is a diagonal matrix such that the first N diagonal entries are equal to λ_{j1} and the remaining diagonal entries are equal to λ_{j2}, with $\lambda_{jj} = 2\nu\kappa P_u$, $\lambda_{j\bar{j}} = 2\epsilon^{-1}(1-\nu)\kappa P_u$ and P_u is the user's average transmit power per channel use. Equation (1) satisfies the uplink power constraint $\mathbb{E}[\|\mathbf{h}_{k,j}\mathbf{\Lambda}_j^{\frac{1}{2}}\|^2] = 2\kappa P_u N$. Thus, the power allocated to feedback the direct and interfering channel is controlled by $\nu \in [0, 1]$. We should note that in (1), it is assumed that κ is an integer. If κN is an integer, we can modulate the signal (1) with $2N \times 2\kappa N$ spreading matrix [5], [6] and the analysis presented below still holds.

Now, let b_ℓ, $\ell = 1, 2, \cdots, 2N$, be the ℓth element of $\mathbf{h}_{k,j}$, λ_ℓ be the corresponding element on the diagonal of $\mathbf{\Lambda}$, and $\epsilon_\ell = \mathbb{E}[b_\ell b_\ell^*]$. When this channel coefficient is transmitted, the signal received by the coordinating BSs is

$$\mathbf{y}_\ell = \sqrt{\lambda_\ell}\begin{bmatrix} \mathbf{1}_N \\ \sqrt{\epsilon}\mathbf{1}_N \end{bmatrix} b_\ell + \mathbf{n}_u = \sqrt{\lambda_\ell}\mathbf{p}b_\ell + \mathbf{n}_u,$$

where $\mathbf{n}_u \in \mathbb{C}^{2N} \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2\mathbf{I})$ is the noise vector at the coordinating BSs and $\mathbf{1}_N$ is a column vector of length N with all 1 entries. Using the fact that the path-gain from the users in cell j to BS $\bar{j}$ is ϵ, the MMSE estimate of b_ℓ becomes

$$\hat{b}_\ell = \sqrt{\lambda_\ell}\epsilon_\ell\mathbf{p}^T\left[\lambda_\ell\epsilon_\ell\mathbf{p}\mathbf{p}^T + \sigma_u^2\mathbf{I}_{2N}\right]^{-1}\mathbf{y}_\ell,$$

and its MMSE is $\sigma_{b_\ell}^2 = \epsilon_\ell - \lambda_\ell\epsilon_\ell^2\mathbf{p}^T\left[\lambda_\ell\epsilon_\ell\mathbf{p}\mathbf{p}^T + \sigma_u^2\mathbf{I}_{2N}\right]^{-1}\mathbf{p}$. We should note that $\{\hat{b}_\ell\}$ are mutually independent. Therefore, we can express $\mathbf{h}_{k,j,i}$ as

$$\mathbf{h}_{k,j,i} = \sqrt{\phi_{k,j,i}}\widehat{\mathbf{h}}_{k,j,i} + \widetilde{\mathbf{h}}_{k,j,i}, \tag{2}$$

where $\widehat{\mathbf{h}}_{k,j,i}$ represents the channel estimate, and $\widetilde{\mathbf{h}}_{k,j,i}$ the channel uncertainty or estimation error, with

$$\delta_{ji} = \begin{cases} \frac{1}{1+\nu\bar{\gamma}_u}, & j = i \\ \frac{\epsilon}{1+(1-\nu)\bar{\gamma}_u}, & j \neq i, \end{cases}, \quad \omega_{ji} = \begin{cases} \frac{\nu\bar{\gamma}_u}{1+\nu\bar{\gamma}_u}, & j = i \\ \frac{\epsilon(1-\nu)\bar{\gamma}_u}{1+(1-\nu)\bar{\gamma}_u}, & j \neq i, \end{cases}$$

where $\bar{\gamma}_u = 2\gamma_u\kappa(1+\epsilon)$, with $\gamma_u = NP_u/\sigma_u^2$. Note that the entries of each vector $\widehat{\mathbf{h}}_{k,i,j}$ and $\widetilde{\mathbf{h}}_{k,i,j}$ are independent and identically distributed (i.i.d) and distributed according to $\mathcal{CN}(0, \omega_{k,i,j})$ and $\mathcal{CN}(0, \delta_{k,i,j})$, respectively. The channel estimates are used by the BSs to construct the precoder. In what follows, let us denote $\widehat{\mathbf{h}}_{k,j} = [\widehat{\mathbf{h}}_{k,j,1}\ \widehat{\mathbf{h}}_{k,j,2}]$, $\widetilde{\mathbf{h}}_{k,j} = [\widetilde{\mathbf{h}}_{k,j,1}\ \widetilde{\mathbf{h}}_{k,j,2}]$ and $\widehat{\mathbf{H}} = [\widehat{\mathbf{h}}_{1,1}^H\ \widehat{\mathbf{h}}_{2,1}^H \cdots \widehat{\mathbf{h}}_{K,1}^H\ \widehat{\mathbf{h}}_{1,2}^H\ \widehat{\mathbf{h}}_{2,2}^H\ \cdots\ \widehat{\mathbf{h}}_{K,2}^H]^H$.

B. Network MIMO

Under network MIMO, both BSs share the channel information and data symbols for all users in the network. Therefore, we may consider the network as a broadcast channel with $2N$ transmit antennas and $2K$ single antenna users. The BSs construct the precoding matrix using their channel estimates. Here, we consider regularized channel inversion precoding, for which the transmitted signal $\mathbf{x}$ can be written as

$$\mathbf{x} = \sum_{j=1}^{2} \sum_{k=1}^{K} c\hat{\mathbf{w}}_{kj} s_{kj},$$

where $\mathbf{w}_{kj} = c\left(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}} + \alpha \mathbf{I}\right)^{-1} \widehat{\mathbf{h}}_{k,j}^H$, α being the "regularization parameter", $s_{kj} \sim \mathcal{CN}(0,1)$ denotes the symbol to be transmitted to user k in cell j and c is a scaling factor ensuring the total power constraint is met with equality.

The SINR attained by the user k in cell j is given by

$$\mathrm{SINR}_{kj} = \frac{\left|\mathbf{h}_{k,j}\left(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}} + \alpha \mathbf{I}\right)^{-1} \widehat{\mathbf{h}}_{k,j}^H\right|^2}{\frac{\sigma_d^2}{c^2} + \left\|\mathbf{h}_{k,j}\left(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}} + \alpha \mathbf{I}\right)^{-1} \widehat{\mathbf{H}}_{k,j}^H\right\|^2}, \quad (3)$$

where $\widehat{\mathbf{H}}_{k,j}$ is obtained from $\widehat{\mathbf{H}}$ by removing the row corresponding to user k in cell j.

C. Coordinated Beamforming

In this scheme, the base stations only share the channel information, so that, for cell j, $\mathbf{x}_j$ can be expressed as

$$\mathbf{x}_j = c_j \sum_{k=1}^{K} \hat{\mathbf{w}}_{kj} s_{kj},$$

where as in the network MIMO case $s_{kj} \sim \mathcal{CN}(0,1)$ denotes the symbol to be transmitted to user k in cell j; $c_j^2 = \frac{P_j}{\sum_{k=1}^{K} \|\hat{\mathbf{w}}_{kj}\|^2}$. We let

$$\hat{\mathbf{w}}_{kj} = \left(\alpha \mathbf{I} + \sum_{(l,m)\neq(k,j)} \widehat{\mathbf{h}}_{l,m,j}^H \widehat{\mathbf{h}}_{l,m,j}\right)^{-1} \widehat{\mathbf{h}}_{k,j,j},$$

which is an extension of regularized zero-forcing to the CBf setup [3]. Note that designing the precoding matrix at BS j requires *local* CSI only (the $\widehat{\mathbf{h}}_{k,i,j}$ from BS j to all users, but not the channels from the other BS to the users). The SINR of user k in cell j is given by

$$\mathrm{SINR}_{kj} = \frac{|\mathbf{h}_{k,j,j}\mathbf{w}_{kj}|^2}{\frac{\sigma_d^2}{c_j^2} + \sum_{(k',j')\neq(k,j)} \frac{c_{j'}^2}{c_j^2} |\mathbf{h}_{k,j,j'}\mathbf{w}_{k'j'}|^2}. \quad (4)$$

III. Large System Results

In this section, we will derive the SINR of the network MIMO and CBf in the large systems limit, i.e., $K, N \to \infty$ with $K/N \to \beta$. We will often refer to β as the cell loading. Moreover, we will also derive the optimal regularization parameter and the optimal ν for each cooperation scheme. Throughout the analysis presented below, we assume $P_1 = P_2 = P$ and denote $\gamma_d = P/\sigma_d^2$. Making use of the symmetry in our system setup, we also denote $\delta_{ii} = \delta_d, \delta_{i\bar{i}} = \delta_c, \omega_{ii} = \omega_d = 1 - \delta_d$ and $\omega_{i\bar{i}} = \omega_c = \epsilon - \delta_c$.

A. Network MIMO

Theorem 1: Let $\rho_M = (\omega_d + \omega_c)^{-1}\alpha/N$ and $g(\beta,\rho)$ be the solution of $g(\beta,\rho) = \left(\rho + \frac{\beta}{1+g(\beta,\rho)}\right)^{-1}$. In the large systems limit, the SINR of network MIMO given in (3) converges in probability to a deterministic quantity given by

$$\mathrm{SINR}_{\mathrm{MCP}}^{\infty} = \gamma_e g(\beta,\rho_M) \frac{1 + \frac{\rho_M}{\beta}(1 + g(\beta,\rho_M))^2}{\gamma_e + (1 + g(\beta,\rho_M))^2}, \quad (5)$$

where the effective SNR γ_e is expressed as

$$\gamma_e = \frac{\omega_d + \omega_c}{\delta_d + \delta_c + \frac{1}{\gamma_d}} = \frac{1 - \delta_d + \epsilon - \delta_c}{\delta_d + \delta_c + \frac{1}{\gamma_d}}.$$

Proof: The large systems analysis for each term in (3) follows the approaches taken in [3] based on the results in [7] or in [8]; due to space limitations, only a sketch of the proof is given. For notational simplicity, let $g = g(\beta,\rho_M)$. The numerator (3) can be shown to converge to $g^2/(1+g)^2$. Similarly, the interference term (the second term in the denominator (3)) converges to

$$\left(g + \rho_M \frac{\partial g}{\partial \rho_M}\right)\left[\frac{1}{(1+g)^2} + \frac{\delta_d + \delta_c}{\omega_d + \omega_c}\right]. \quad (6)$$

We can also show that c^2 converges to $P(\omega_d + \omega_c)\left(g + \rho_M \frac{\partial g}{\partial \rho_M}\right)^{-1}$. Combining these results yields (5) after some algebraic manipulations. ■

It is obvious from (5) that based on our assumptions, the limiting SINR is the same for all users in both cells. Since ρ_M controls the amount of interference introduced to the users, its choice is important in the system design. Following [9], it can be shown that the optimal ρ_M that maximizes $\mathrm{SINR}_{\mathrm{MCP}}^{\infty}$ is

$$\rho_M^* = \frac{\beta}{\gamma_e}, \quad (7)$$

and the corresponding limiting SINR is

$$\mathrm{SINR}_{\mathrm{MCP}}^{*,\infty} = g(\beta,\rho_M^*).$$

Since g is decreasing in ρ_M then for fixed β the limiting SINR is maximized by maximizing γ_e. In order to obtain the optimal power allocation for feeding back the direct and interfering CSI, we thus pose the following optimization problem

$$\max_{\nu\in[0,1]} \quad \gamma_e = \frac{\epsilon - \delta_c + 1 - \delta_d}{(\delta_d + \delta_c) + \frac{1}{\gamma_d}}.$$

It is obvious from the effective SNR expression that it is a decreasing function of $\delta_c + \delta_d$. Therefore, we can rewrite the optimization problem above as

$$\min_{\nu\in[0,1]} \quad \delta_t = \delta_d + \delta_c = \frac{1}{\nu\bar{\gamma}_u + 1} + \frac{\epsilon}{(1-\nu)\bar{\gamma}_u + 1}.$$

From the above, it is very interesting to note that *the optimal ν that maximizes $SINR_{MCP}^{*,\infty}$ is the same as the one that minimizes the total of mean square error (MSE), δ_t.*

It is easy to check that the optimization problem above is a convex program and the optimal ν^* can be shown to be

$$\nu^* = \begin{cases} 0, & \sqrt{\epsilon} \geq \bar{\gamma}_u + 1 \\ 1, & \sqrt{\epsilon} \leq \frac{1}{\bar{\gamma}_u + 1} \\ \frac{1 + \frac{1}{\bar{\gamma}_u}(1-\sqrt{\epsilon})}{1+\sqrt{\epsilon}}, & \text{otherwise.} \end{cases} \quad (8)$$

As a result, for $\sqrt{\epsilon} \leq \frac{1}{\bar{\gamma}_u + 1}$, the BSs should not waste resources trying to learn about the "interfering" channel states. In this situation, the coordination breaks down and the base stations perform SCP. The completely opposite scenario, in which the BSs should not learn the "direct" channels, occurs when $\sqrt{\epsilon} \geq \bar{\gamma}_u + 1$. Clearly, this can only happen if $\epsilon > 1$. When $\sqrt{\epsilon} \geq \bar{\gamma}_u + 1$, the BSs also perform SCP but each BS transmits to the users in the neighboring cell.

We end this section by characterizing the behavior of γ_e (equivalently $\text{SINR}_{\text{MCP}}^{*,\infty}$), after optimal feedback power allocation, as the cross gain ϵ varies. This also implicitly shows how the total MSE, δ_t, affects the limiting SINR.

Let $\breve{\gamma}_u = \frac{\bar{\gamma}_u}{(1+\epsilon)}$. We analyze the different cases in (8) separately.

1) $\sqrt{\epsilon} \leq \frac{1}{\bar{\gamma}_u+1}$: This is the case when the BSs perform SCP for the users in their own cell. For fixed $\breve{\gamma}_u$, this inequality is equivalent to $\epsilon \leq \epsilon_{\max}^{\text{SCP}}$, where $\epsilon_{\max}^{\text{SCP}} \geq 0$ satisfies $\sqrt{\epsilon_{\max}^{\text{SCP}}} = \frac{1}{\breve{\gamma}_u(1+\epsilon_{\max}^{\text{SCP}})+1}$. Now, by taking the first derivative $\frac{\partial \gamma_e}{\partial \epsilon}$ and setting it to zero, the (unique) stationary point is given by

$$\epsilon_{\text{AF}}^{\text{SCP}} = \frac{1}{\sqrt{\gamma_d \breve{\gamma}_u}} - 1.$$

If $\sqrt{\epsilon_{\text{AF}}^{\text{SCP}}} \in [0, \sqrt{\epsilon_{\max}^{\text{SCP}}}]$, it is easy to check that the limiting SINR is increasing until $\epsilon = \epsilon_{\text{AF}}^{\text{SCP}}$ and then decreasing. If $\sqrt{\gamma_d \breve{\gamma}_u} > 1$ then $\epsilon_{\text{AF}}^{\text{SCP}} < 0$, or equivalently, $\frac{\partial \gamma_e}{\partial \epsilon} < 0$. Consequently, the limiting SINR is decreasing in ϵ in this case. Moreover, $\sqrt{\epsilon_{\text{AF}}^{\text{SCP}}} \geq \sqrt{\epsilon_{\max}^{\text{SCP}}}$ if the following condition holds

$$\sqrt{\gamma_d \breve{\gamma}_u}(2 - 2\gamma_d - \breve{\gamma}_u) \geq (2\gamma_d \breve{\gamma}_u - \gamma_d - \breve{\gamma}_u), \quad (9)$$

in which case $\frac{\partial \gamma_e}{\partial \epsilon} > 0$, which implies that the limiting SINR always increases over ϵ for this case.

This behavior of γ_e as a function of ϵ can be intuitively explained as follows. When $\nu = 1$, the total MSE is $\delta_t = \frac{1}{(1+\epsilon)\breve{\gamma}_u + 1} + \epsilon$, where the first and second terms are δ_d and δ_c, respectively. As ϵ increases, δ_d decreases whereas δ_c increases. This shows that there is a trade-off between the quality of the direct channel and the strength of the interference. The trade-off is also influenced by parameters γ_d and $\breve{\gamma}_u$. As shown in the analysis, when $\sqrt{\gamma_d \breve{\gamma}_u} > 1$, the effect of cross channel to the limiting SINR dominates. In contrast, if the condition in (9) is satisfied, the effect of the quality of the direct channel (δ_t) becomes dominant. If the aforementioned conditions do not hold, δ_t causes the SINR to increase until $\epsilon_{\text{AF}}^{\text{SCP}}$ and after that the interference from the cross channel takes over as the dominant factor, thereby reducing the limiting SINR.

2) $\bar{\gamma}_u + 1 \geq \sqrt{\epsilon} \geq \frac{1}{\bar{\gamma}_u+1}$: Here, the BSs perform MCP. By taking $\frac{\partial \gamma_e}{\partial \epsilon}$ in that interval of ϵ, it can be shown that we have a unique stationary which we denote as $\sqrt{\epsilon_{\text{M}}}$. We can also show that γ_e is a convex function for $\epsilon \in [0, 1]$ and is increasing for $\epsilon \geq 1$. Thus, if $\frac{1}{\bar{\gamma}_u+1} \leq \sqrt{\epsilon_{\text{M}}} \leq \bar{\gamma}_u + 1$, the limiting SINR will decrease for $\sqrt{\epsilon} \in [\frac{1}{\breve{\gamma}_u(1+\epsilon)+1}, \sqrt{\epsilon_{\text{M}}}]$ and increase after that; Otherwise, the limiting SINR increases in the region. Here, for $\sqrt{\epsilon} \in [\frac{1}{\bar{\gamma}_u+1}, 1]$, we still can see the effect of the trade-off within δ_t to the limiting SINR as ϵ changes. In that interval, the quality of the direct channel becomes better as ϵ increases; However, that of the cross channel decreases and this affects the SINR badly until ϵ_{M}. After this point, the improvement in the quality of the direct channel will outweigh the deterioration of that of the cross channel, causing the SINR to increase.

3) $\sqrt{\epsilon} \geq \bar{\gamma}_u + 1$: In this case, each BS performs SCP, but serves the other cell's users. We can establish that $\frac{\partial \gamma_e}{\partial \epsilon} > 0$. Hence, the limiting SINR is increasing in ϵ in this region.

B. Coordinated Beamforming

Theorem 2: Let $\rho_C = \frac{\alpha}{N}$, and let Γ be the solution of the following cubic equation

$$\Gamma = \frac{1}{\rho_C + \frac{\beta\omega_c}{1+\omega_c\Gamma} + \frac{\beta\omega_d}{1+\omega_d\Gamma}}. \quad (10)$$

In the large systems limit, the SINR for coordinated beamforming as given by (4) converges almost surely to a deterministic quantity given by

$$\text{SINR}_{\text{CBf}}^{\infty} = \frac{\frac{\omega_d}{\beta}\Gamma\left[\rho_C + \frac{\beta\omega_c}{(1+\omega_c\Gamma)^2} + \frac{\beta\omega_d}{(1+\omega_d\Gamma)^2}\right]}{\left(\frac{1}{\gamma_d} + \delta_d + \delta_c + \frac{\omega_d}{(1+\omega_d\Gamma)^2} + \frac{\omega_c}{(1+\omega_c\Gamma)^2}\right)}. \quad (11)$$

Proof: Here, we only present the large systems results for the terms in (4). The details follow along the similar techniques to those in [3]. The numerator (4) converges almost surely to $\omega_d^2\Gamma^2$. The term c^2 converges almost surely to $-P\left(\beta\omega_d\frac{\partial\Gamma}{\partial\rho_C}\right)^{-1}$. Since $c_j = c_{\bar{j}}$ in the large systems limit, the interference term converges almost surely to

$$-\beta\omega_d\left(\frac{\omega_d}{(1+\omega_d\Gamma)^2} + \frac{\omega_c}{(1+\omega_c\Gamma)^2} + \delta_d + \delta_c\right)\frac{\partial\Gamma}{\partial\rho_C}.$$

Putting these results together, and differentiating (10) to get an expression for $\frac{\partial\Gamma}{\partial\rho_C}$, we get (11). ■

The optimal ρ_C that maximizes the limiting SINR (11) is given in Corollary 1. By comparing (7) and (12), it is interesting to see that $\rho_C = \rho_M$ for a given α.

Corollary 1: The limiting SINR (11) is maximized by choosing the regularization parameter according to

$$\rho_C^* = \beta\left(\frac{1}{\gamma_d} + \delta_d + \delta_c\right). \quad (12)$$

and the corresponding limiting SINR is

$$\text{SINR}_{\text{CBf}}^{*,\infty} = \omega_d\Gamma. \quad (13)$$

Proof: Let $\gamma_{eff} = \beta\left(\gamma_d^{-1} + \delta_d + \delta_c\right)$ and $\Psi(\rho_C) = \frac{\omega_d}{(1+\omega_d\Gamma)^2} + \frac{\omega_c}{(1+\omega_c\Gamma)^2}$. It is easy to show that

$$\frac{\partial\text{SINR}_{\text{CBf}}^{\infty}}{\partial\rho_C} = \omega_d\Psi'(\rho_C)\frac{\gamma_{eff} - \rho_C}{[\gamma_{eff} + \Psi(\rho_C)]^2}. \quad (14)$$

It is obvious from (14) that the stationary point and the global optimizer is $\rho_C^* = \gamma_{eff}$. Using this ρ_C in (11) yields (13). ■

Finding ν that maximizes the limiting SINR is more complicated than in the network MIMO case. From Corollary 1, it is equivalent to maximizing $\omega_d \Gamma$, such that $\nu \in [0, 1]$: this is a non-convex program. However, we can show that the maximizer ν^* is one of three points: the boundaries of the feasible set ($\nu = \{0, 1\}$) or the stationary point which is the solution of

$$\nu = -\frac{\Gamma}{\Gamma'_\nu (1 + \nu \bar{\gamma}_u)}. \tag{15}$$

The first possibility $\nu = 0$ can be eliminated as an optimal solution candidate since the derivative of the limiting SINR with respect to ν at this point is always positive.

IV. Numerical Results

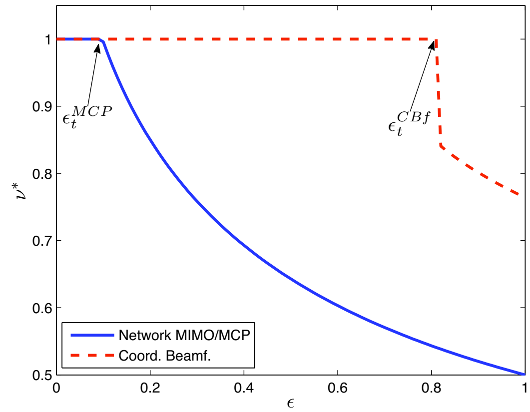
In this section, we present some numerical simulations to visualize the characteristics of the optimal ν^* and the corresponding limiting SINR for each cooperation scheme. We are primarily interested with their characteristics when the interfering channel gain ϵ varies, as depicted in Figure 2.

In general, we can see that for the same system parameters, the CBf scheme allocates more power to feed back the direct channel compared to network MIMO. From Figure 2(a), we can see that for values of ϵ ranging from 0 up to a certain threshold ϵ_t, which is different for each scheme, the optimal ν is 1: in other words, it is optimal in this range for the BSs not to try to get information about the cross channels and to construct the precoder based on the direct channel information only. Effectively, the two schemes reduce to the SCP scheme when $\nu^* = 1$: as a result, the same limiting SINR is achieved by both schemes.

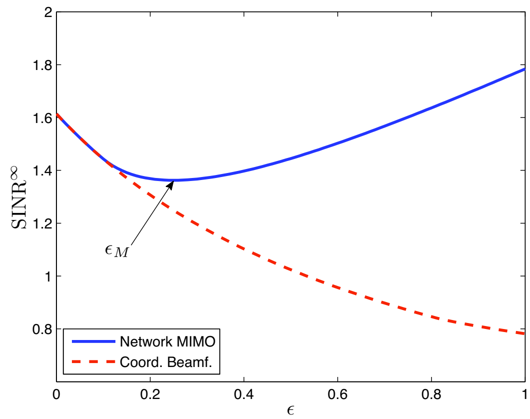
We also observe a peculiar behavior of the limiting SINR of network MIMO, which we already highlighted in the analysis of Section III-A. When $\sqrt{\epsilon} \leq \frac{1}{\bar{\gamma}_u + 1}$, i.e. when $\nu^* = 1$, the SINR is decreasing as ϵ increases. After that the SINR is still decreasing until ϵ reaches ϵ_{M} and then increasing: this reflects the trade-off between δ_c and δ_d. Note that this initial decrease does not occur in the perfect CSI case where the SINR is strictly increasing in ϵ for network MIMO. Similar to the network MIMO case, we can see that the limiting SINR of CBf is decreasing in ϵ when $\nu^* = 1$ (SCP). Moreover, it is still decreasing when both BSs perform CBf.

V. Conclusion

In this paper, we used random matrix theory to analyze a simple analog feedback scheme and its effects on performance in a two-cell setup, for regularized zero-forcing versions of network MIMO and coordinated beamforming. For both schemes, we showed the existence of a threshold value of the cross channel gain ϵ below which it is better for the BSs not to learn the interfering channel and to simply perform single-cell processing. Moreover, for network MIMO, whereas in the full CSI case at the BSs, SINR increases with ϵ, when feedback is limited, this behavior only appears for ϵ above a certain value.



(a)



(b)

Fig. 2. (a) The optimal ν^* and (b) SINR_∞ for the network MIMO and CBf scheme as ϵ varies in $[0, 1]$ with $\beta = 0.6$, $\gamma_d = 10$ dB, $\gamma_u = 0$ dB.

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